

VERTICAL, UPWARD GAS-LIQUID TWO-PHASE FLOW ACROSS A TUBE BUNDLE

R. ULBRICH[†] and D. MEWES

Institut für Verfahrenstechnik, University of Hannover, 30167 Hannover, Germany

(Received 7 October 1992; in revised form 31 August 1993)

Abstract—The vertical two-phase cross-flow was studied on the shell side of a horizontal tube bundle. The section used consists of 5 tubes with 20 mm dia. The tubes are arranged in 10 rows on a square pitch with a pitch ratio of 1.5 in a rectangular shell. Adiabatic flows of air-water mixtures were tested in a large superficial velocity range, for liquid (0.001-0.65 m/s) and for gas (0.047-9.3 m/s), respectively. Flow patterns were established and visual observation together with photographic data and a video-film were used. These results, existing flow pattern data and flow pattern maps for the cross-flow in tube bundles were employed to work out a general flow pattern map.

Key Words: two-phase flow, gas-liquid, vertical, cross-flow, tube bundle, flow pattern, flow regime map

INTRODUCTION

As opposed to gas-liquid two-phase flow in tubes, which is rather well examined, investigations concerning the flow on the shell sides are incomplete. Such a case is often met when the phase change (boiling, condensation) occurs on the shell side of a shell-and-tube heat exchanger. Then the exchangers play the parts of boilers, steam generators and nuclear reactor coolers. The hydrodynamics of gas-liquid two-phase flow on the shell side of a heat exchanger was examined at the NEL Laboratories (Sutherland & Murray 1969; Grant & Murray 1972, 1974; Grant & Chisholm 1979). Finally, Chisholm collected a great deal of data and presented them in a monograph (Chisholm 1983).

Although Ishihara *et al.* (1980) have tackled the problem of calculating the pressure drop, interest in two-phase flow on the shell side has only become evident recently, e.g. the works by Schrage *et al.* (1988) and Dowlati *et al.* (1990a, b).

Special attention should be paid to the general tendency to refer the methods of calculation for the pressure drop and gas void fraction, as well as the methods determining the flow patterns for the flow on the shell side, to two-phase flow in pipes. The reader should be warned, however, that it is problematic to transfer these dependences simply. For example, for flow in a pipe, according to the Lockhart & Martinelli (1949) method or the Muller-Steinhagen & Heck method (1986), the two-phase mixture pressure drops are determined on the basis of the single-phase pressure drops when either phase flows separately. Single-phase fraction factors are computed for two distinctly separated regions (laminar and turbulent flows) from the simple dependence

$$\lambda = C \operatorname{Re}^{n},$$
^[1]

where C is constant, n is a power and Re is the Reynolds number. For laminar flow, C = 64, n = -1; whereas for turbulent flow, C = 0.3164, n = -0.25. A rough analysis of the dependence $\zeta = f(\text{Re})$ for tube bundle cross flow (VDI 1977; HEDH 1983) has already shown that this is a case which is far too complicated and too difficult to be described simply.

If, for the one-phase flow in the staggered arrangement, the exponent in the equation

$$\zeta = C \operatorname{Re}^n$$
[2]

changes monotone from -1 to 0, then for the in-line tube arrangement, particularly in the region where the Reynolds number occurs the most frequently (Re = 10^2 to 2.10⁴), it is difficult to discover

^{*}Permanent address: Heat Technique and Chemical Engineering Department, Opole Technical University, ul. Mikołajczyka 5, 45-233 Opole, Poland.

any regularities. Consequently, it is controversial to apply the regular dependence to the Lockhart-Martinelli parameter

$$X_{\rm LM} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_{\rm G}}{\rho_{\rm L}}\right)^{0.5} \left(\frac{\eta_{\rm L}}{\eta_{\rm G}}\right)^{0.3}$$
[3]

for cross-flow across a tube bundle, as it is obligatory only when the exponent in [1] is equal to n = -0.2. In [3], x is the gas mass ratio, ρ_G and ρ_L are the density of the gas and liquid and η_G and η_L are the dynamic viscosity of the gas and liquid, respectively.

The possible cases of two-phase mixture flow on the shell side are supplemented by the flow along the tube bundle, which takes place under certain conditions, and also when the shell side is provided with cross-segmental baffles and longitudinal flow is accomplished in the baffle window. Investigations on flow patterns in the flow along a tube bundle were conducted by Bergles *et al.* (1968), Williams & Peterson (1978), Venkateswararao *et al.* (1982) and Haquet *et al.* (1991). The flow along a tube bundle concerns the flow across a complex geometry cross-section and yet it is constant and invariable along the channel length. The flow pattern map, worked out by Venkateswararao *et al.* (1982) for flow along a tube bundle, which shows good agreement with various experimental data, can be accepted as satisfactory.

In contrast, for tube bundle cross-flow, whose cross-section may be assumed as a rectangle with the sides constituting the length of the tube and a gap between the tubes, its surface (in-line arrangement) and direction (staggered arrangement) can change with great frequency.

Tube bundle cross-flow is also observed in bubble or fluid columns when heat exchanger tube bundles are introduced into traditionally empty tubes (in which gas bubbles flow through stagnant or circulating liquid), in order to supply or receive heat energy (Korte *et al.* 1988; Steiff *et al.* 1991). The range of phase velocities in this kind of tube bundle arrangement differs completely from that in two-phase flow in heat exchangers in the phase change process. By way of analogy to two-phase flow in a pipe, we observe one-component flow (e.g. water-steam, where velocities are limited by the heat exchange process) or two-component flow (e.g. water-air where velocities result only from the quantity of fluxes conveyed to the apparatus).

Recent results on tube or tube bundle vibrations induced by tube bundle two-phase cross-flow are given by Hara (1984, 1987, 1988), Pettigrew *et al.* (1988) and Jatzlau (1990). These studies are in connection with the problem of excitation of tube bundle vibrations and the possibility of damage occurring which may be dangerous in some applications (nuclear power engineering). So far, all works on this subject have considered a homogeneous flow model which is imprecise and which, from the point of view of the hydrodynamics of multiphase flow, should be treated as a first rough approximation.

The first map for tube bundle cross-flow was presented by Grant & Murray (1972) (figure 1). The investigations were conducted with a model which was a rectangular shell, in which 39 tubes, each of dia 19 mm, were placed in a staggered arrangement with a pitch equal to 1.25 tube diameters. Three segmental baffles give four passes. On the basis of visual observations, three flow patterns were distinguished: dispersed (spray), bubbly and intermittent flows. In the work by Grant & Murray (1972) the results were presented in the arrangement proposed by Baker (1954), however, since the work by Grant (1973) the flow pattern map has commonly been presented in the coordinate system, as modified by Bell *et al.* (1970):

$$u_{sG}^* = u_{sG} \left(\frac{\rho_G}{\rho_L}\right)^{0.5}$$
[4]

and

$$u_{sL}^{*} = u_{sL} \frac{(\rho_L \eta_L)^{0.33}}{\sigma}, \qquad [5]$$

where σ is the surface tension. The narrow range of application of the map is remarkable: for liquid velocities this range (maximum to minimum ratio) is 10, whereas for gas it is 20. Kondo & Nakajima (1980) studied the narrow range of application of the map proposed by Grant & Murray (1972) and found that it may not be used to analyze hydrodynamic phenomena occurring in thermosiphon reboilers. Also, in the case of fluid columns, the range of their work is outside the range of application of the map presented by Grant & Murray (1972).

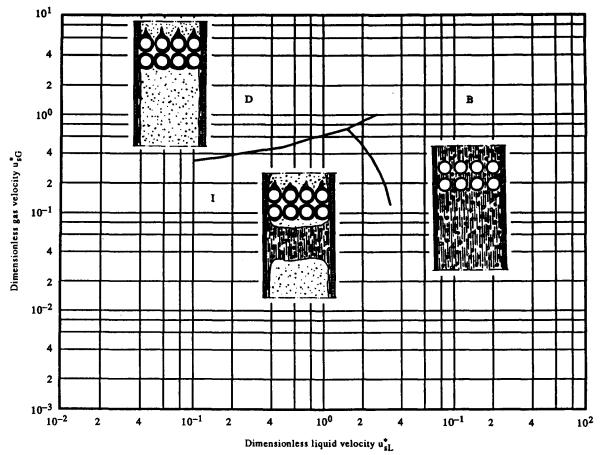


Figure 1. Flow patterns and the flow pattern map for vertical air-water two-phase flow across a tube bundle proposed by Grant & Murray (1972) (B-bubble, I-intermittent, D-dispersed flows).

Superficial velocities occurring on the coordinate axes should, in the case of tube bundle cross-flow, be treated as the velocities which have reference to the overall cross-section flow but not to the region occupied by a given phase at any one moment. The minimum cross-sectional area between tubes is generally assumed as the cross-section flow.

The map proposed by Grant & Murray (1972) was also used by Pettigrew et al. (1988) after transposing it into the coordinate system

$$X_{\text{LM,hom}} = \left(\frac{1-\epsilon}{\epsilon}\right)^{0.9} \left(\frac{\rho_{\text{L}}}{\rho_{\text{G}}}\right)^{0.4} \left(\frac{\eta_{\text{L}}}{\eta_{\text{G}}}\right)^{0.1}$$
[6]

and

$$u_{sG}^{**} = \frac{\rho_G u_{sG}}{[d_h g \rho_G (\rho_L - \rho_G)]^{0.5}},$$
[7]

where u_{sG} is the superficial gas velocity, g is the acceleration of gravity, ϵ is the homogeneous gas void fraction and d_h is the hydraulic diameter, interpreted as a double gap between tubes

$$d_{\rm h} = 2(t-d). \tag{8}$$

The boundary between bubble and intermittent flow was also determined by Pettigrew *et al.* (1988). They propose that, for $\epsilon = 0.9$, the transition from bubble to intermittent flow takes place. It seems that this value is greatly overestimated. For water-air flow the homogeneous void fraction $\epsilon = 0.9$ corresponds to the gas void fraction determined by the Stomma (1979) method $\epsilon = 0.7$. The value is considerably overestimated if it concerns two-phase flow in a pipe, where $\epsilon = 0.50$ to 0.52 is generally assumed for the transition boundary from bubbly to intermittent flow.

Jatzlau (1990) presented the results of his own research for a one-component Freon-12 mixture two-phase flow under high pressure in the arrangement modified by Pettigrew *et al.* (1988), and he found good agreement for the transition between bubble and intermittent flow.

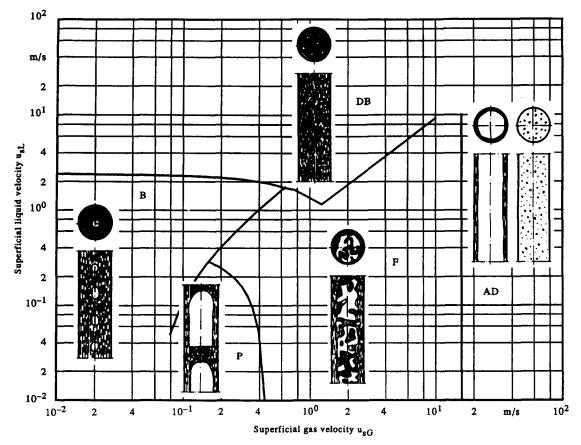


Figure 2. Flow pattern map for vertical upward gas-liquid two-phase flow in a tube proposed by Taitel *et al.* (1980); d = 20 mm, P = 1 bar, $T = 20^{\circ}\text{C}$ (B--bubble, DB--elongated bubble, P--plug, F--frothy, A--annular, D--dispersed flows).

Many attempts have already been mentioned which attempted to correlate the results of investigations for tube bundle cross-flow by using methods valid for flow in a pipe. A fact which encourages this procedure is the successful attempt to adapt a theoretical model of flow pattern maps proposed by Taitel *et al.* (1980) (figure 2) for longitudinal flow along a tube bundle (Venkateswararao *et al.* 1982).

It seems that it is possible to assume, as an extreme simplification, that tube bundle cross-flow occurs in a rectangular channel with hydraulic diameter d_h and with velocities concerning the section between the tubes. In such a situation, the model proposed by Taitel *et al.* (1980) is worthy of consideration, but one must take into account the fact that some flow patterns in a pipe do not appear in tube bundle cross-flow.

The second extreme case, considering rather large dimensions in the channel cross-section, is the treatment of the entire apparatus as a fluid column free from a tube bundle. Such flow conditions exist before and behind the tube bundle. A very interesting analysis of such a two-phase flow is given in the work by Sollychin *et al.* (1991), where two-phase flow in a bubble column is analysed, for the first time, under conditions different from typical bubble columns (liquid is the continuous phase).

Results of the investigations of two-phase flow patterns concerning tube bundle cross-flow are presented in table 1; data which concern cross-flow in tube bundles of an archaic form (a single tube, a tube row or a series of single tubes) are also listed. Practically all the investigations concern a water-air mixture and the majority refer to bubble flow. In figure 3 the results of the investigations listed in table 1 are compared with the flow pattern map proposed by Grant & Murray (1972). A significant number of points lie outside the range of the transition lines proposed by these authors, which compels one to extrapolate the lines—frequently in a perilous manner.

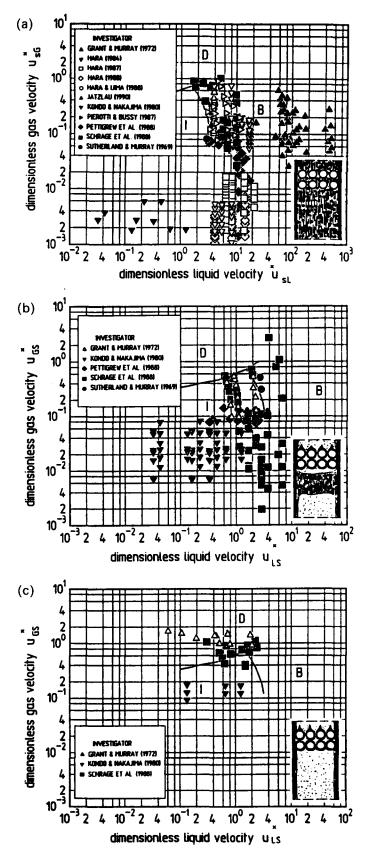


Figure 3. Comparison of the data presented in table 1 with the flow pattern map of Grant & Murray (1972): (a) bubble flow; (b) intermittent flow; (c) dispersed flow.

		Ċ	Oriontotion ^a	a se		Tube bundle	ndle		H	Fluid mixture			
No.	Investigator(s)	2 Ph	Tube	ء 10	[mm]	[mm]	ш	r		g_{T} [kg/(m ² s)]	x	Flow regime ^c	No. of data
	Sutherland & Murray (1969)	ΝŪ	H		19.0		5	-	Water-air p = 1.15 bar $\cdot - 2 \div 0.11 \circ C$	200-886	0.00035-0.091	B, I	s
7	Grant & Murray (1972)	ΝU	Н	sн	19.0	1.25	5	4	p = 1.03 to 1.00 Water-air p = 1.03 to 1.65 bar	58-1124	0.0030-0.94	B, I, D	56
ŝ	Grant & Murray (1974)	Н	Н	S T	19.0	1.25	5	4	f = 4 to 20 C Water-air p = 1.01 to 1.66 bar	17–1124	0.0020-0.93	S, B, I, D	2
4	Kondo & Nakajima (1980)	٧U	Н	sF	25.0	80.1	7–20	S	$\begin{array}{l} p = 1.20 \text{ bar} \\ p = 1.20 \text{ bar} \\ p = 1.00 \text{ cm} \end{array}$	2.4-90	0.00070-0.47	B, P, F, D	20
S	Kondo & Nakajima (1980)	ΝU	Н	s T	25.0	1.28	7-20	5	p = 13 C Water-air p = 1.20 bar r = 100C	3.2–35	0.006-0.47	B, P, F	21
9	Kondo & Nakajima (1980)	٧U	Н	s F	25.0	1.40	7–20	S	p = 1.02 bar p = 1.02 bar	2.4-25	0.003-0.44	B, P, F	22
٢	Hara (1984)	٧U	Н	1	30.0	ļ	1	5	p = 19 C Water-air $p = 1.0 bar$	300-600	0.000015-0.0014	B	70
œ	Hara (1987)	ΝU	Н	I	25.0	1.33	-	5	$p = \frac{1}{2000}$ Water-air p = 1.0 bar	3001530	0.000015-0.0014	B	122
6	Pierotti & Bussy (1987)	ΝU	Н	S Q	25.0	1.44	14	٢	p = 20 C Water-R13B1 p = 7.5 bar	270-900	0.03-0.375	в	15
10	Pierotti & Bussy (1987)	٧U	H	ЧС	25.0	1.44	14	٢	p = 7.5 bar p = 7.5 bar	540	0.049-0.375	в	7
Ξ	Hara & Ijima (1988)	νυ	Η	l	25.0	1.33	_	S	$W_{ater-air} = 1.0 \text{ bar}$ $y = 20\% \text{ C}$	170-1165	0.0000400.0023	В	87
12	Hara (1988)	٧U	Ξ	-	25.0	3.0	C I	_	Water-air p = 1.0 bar $t = 20^{\circ}$ C	300-1060	0.000060-0.00038	В	56

R. ULBRICH and D. MEWES

13	Pettigrew <i>et al.</i> (1988)	ΝŪ	Н	sT	13.0	1.32	10	٢	Water-air p = 1 bar t = 20°C	240-1012	0.000066-0.016	B, I	12
14	Pettigrew <i>et al.</i> (1988)	ΝŪ	Н	s F	13.0	1.47	10	٦	Water-airp = 1 barr = 20°C	45-1200	0.0004-0.11	B, I	12
15	Pettigrew <i>et al.</i> (1988)	٧U	н	ЧГ	13.0	1.47	10	٢	$Water-air p = 1 \text{ bar} r = 20^{\circ}\text{C}$	28688	0.000066-0.11	B, I	15
16	Pettigrew <i>et al.</i> (1988)	νυ	Н	ЧС	13.0	1.47	01	٢	Water-air	29-1013	0.000066-0.11	B, I	16
17	Schrage <i>et al.</i> (1988)	ΝŪ	Н	ЧQ	7.9	1.30	27	S	$water-airp = 1 to 3 bart = 20^{\circ}C$	58-683	0.00060-0.65	B, I, D	82
18	Jatzlau (1990)	٨U	Н	ЧL	22.0	1.50	1-5	4	R12-vapour $p = 10 to 32 bar$	270–1880	0.0085-0.17	В	65
= ^ =	${}^{a}V = vertical, H = horizontal, U = upward, D = downward$	l, U = u	ıpward	, D = dc	wnward.								

 $^{b}L = in-line$, S = staggered. $^{c}B = bubbly$, I = intermittent, D = dispersed, S = stratified, P = plug, F = frothy.

Table 2. Statistical analysis of flow pattern predictions for	r the data described in table 1: (a) Grant & Murray (1972); (b) Grant & Murray (1972) with restrictions; (c) Taitel et al. (1980)	& Murray (1972) with restrictions; (c) Taitel et al. (1980)
(a) Data: total	(b) Data: total	(c) Data: total
Flow reg. map: Grant & Murray (1972)	Flow reg. map: Grant & Murray (1972)	Flow reg. map: Taitel et al. (1980)
Exp. flow pattern	Exp. flow pattern	Exp. flow pattern
No. of data points B I D Sum	No. of data points B I D Sum	No. of data points B I D Sum
Pred. flow pattern B 167 7 1 175	Pred. flow pattern B 56 6 1 63	Pred. flow pattern B 201 13 1 215
D 1 341 119 11 4/1 D 1 1 19 21	D 1 0 49 D 1 0 9 10	$\begin{array}{cccccccccccccccccccccccccccccccccccc$
UN 0 0 0 0	446 88 11	UN 0 0 0 0
Sum 509 127 31 667	Sum 509 127 31 667	Sum 509 127 31 667
45.7% of data points are predicted correctly	14.7% of data points are predicted correctly	49.0% of data points are predicted correctly

VERTICAL TWO-PHASE CROSS-FLOW IN A TUBE BUNDLE

.

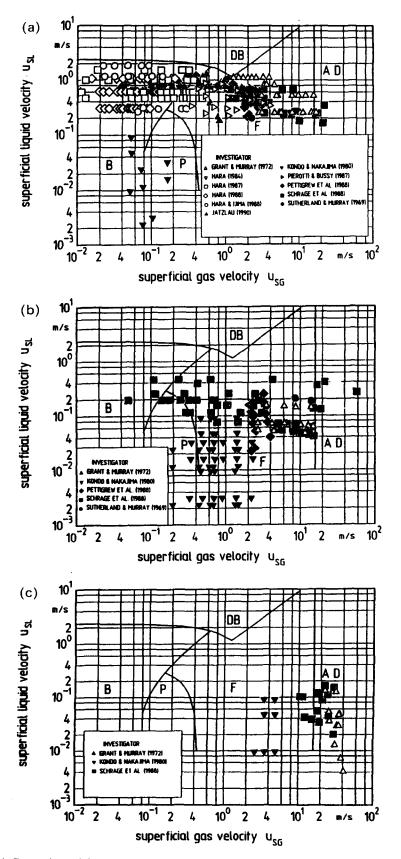


Figure 4. Comparison of the data presented in table 1 with the flow pattern map of Taitel *et al.* (1980): (a) bubble flow; (b) intermittent flow; (c) dispersed flow.

The results of the statistical calculations of the agreement between the flow patterns observed and those plotted on a flow pattern map are presented in tables 2(a) and 2(b). In the case of the extrapolation of boundary lines in a manner very similar to that applied by Schrage *et al.* (1988), nearly 46% of the points show agreement between the observed and plotted flow patterns [table 2(a)]. In the case of application of the flow pattern map proposed by Grant & Murray (1972) with some restrictions (restriction to the area tested by these authors), only about 15% of the data from the total number show agreement, whereas almost 80% of the points appearing in the area tested by Grant & Murray [in table 2(b) excepting points UN—unknown flow pattern] show agreement between the observations and the flow pattern map. This distinctly confirms that the extrapolation of boundary lines outside the tested region is very risky.

In the case of the map proposed by Taitel *et al.* (1980) (figure 2), after taking into account the fact that flow patterns DB and B occur as bubble flow pattern, flow patterns P and F occur as intermittant flow pattern and flow pattern AD occurs as a dispersive flow pattern, the area in which an intermittent flow pattern occurs [figure 4(b)] is relatively well-mapped. Almost 50% of the points show agreement between the flow observed and that plotted on the flow pattern map [table 2(c)].

Thus, based on the data presented in table 1, the flow pattern map of Grant & Murray (1972) (figure 1), for tube bundle cross-flow, is not statistically better than the flow pattern map of Taitel *et al.* (1980), for flow in a pipe.

The facts presented above encouraged the present authors to investigate the range of occurrence of two-phase flow patterns for tube bundle cross-flow.

EXPERIMENTAL APPARATUS

The flow apparatus was designed to allow adiabatic flow experiments with air-water mixtures. The apparatus is illustrated schematically in figure 5. Air is supplied from a compressed air storage tank and flows through a pressure regulator and a preselected rotameter to an air-water mixing chamber. The air flow rate was controlled by a valve located at the inlet of the rotameter and

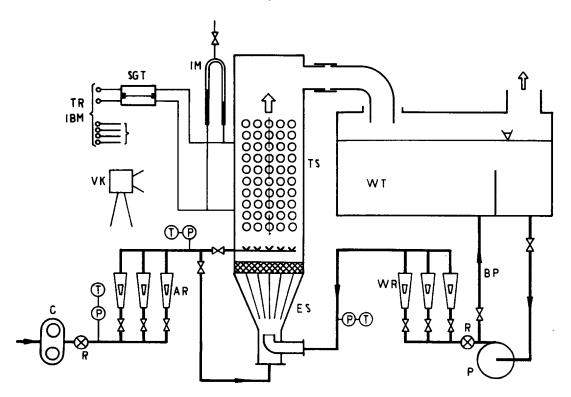


Figure 5. Schematic diagram of the experimental apparatus (TS—test section, ES—mixing chamber, WT—water tank, P—pump, C—compressor, AR—air rotameters, WR—water rotameters, VK—video-camera, BP—bypass, SGT—strain gauge transducer, IM—inverted manometer, R—regulator, TR—transient recorder, IBM—personal computer, T—temperature sensor, P—pressure sensor).

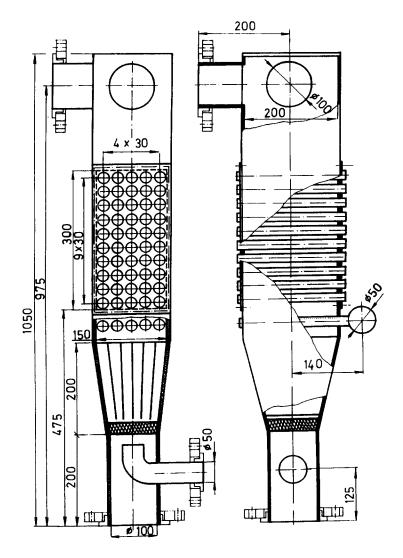


Figure 6. Test-section assembly.

the air pressure was monitored with a Bourdon gauge. Water stored in a tank was pumped through rotameters into the mixing chamber at the bottom of the test section. The two-phase mixture travelled through the mixing chamber and transparent vertical rectangular test section $(150 \times 200 \text{ mm} \text{ and } 650 \text{ mm} \text{ long})$ containing the rod bundle and passed through a 100 mm i.d. nozzle into an open-top separator tank. Air went to the atmosphere and water returned to the reservoir.

The tube bundle, illustrated in figure 6, consists of 10 rows of 5 tubes, each 200 mm and 20 mm o.d. and arranged in line with a pitch of 30 mm. Air was injected through the inlet nozzle (100 mm i.d.) or through 5 tubes: 200 mm long and 20 mm o.d. with 60 holes of 2 mm i.d. at the bottom of the tubes. The 5 tubes were arranged in line on the plane. The clearance of tubes was equal to the tube gap in the bundles. Water was fed through a bend with 50 mm i.d. The mixing section consisted of inlet nozzles at the entrance to the test section and a series of 5 wire meshes as a tranquilization screen. Flow straightners, in the form of slightly divergent, 150 mm long plates, were located above the screen and 75 mm from the bottom of the tube bundle. The top of the tube bundle was located approx. 300 mm below the exit nozzle. The air and water temperatures in the flowmeter and in the reservoir were measured with thermometers based on the electrical conductivity. The water temperature in the reservoir was adopted as the characteristic temperature of the air–water mixtures, since the difference in the water temperatures between the reservoir and the separator was <1°C.

In order to obtain pressure drop data free of end effects, a differential pressure transducer (Sensotec model TJE with a range of 0-10 kPa) was connected to the pressure taps located above the second and below the ninth row, respectively. All pressure-sensing lines were purged free of air bubbles before each test run and the signals were stored. The pressure drop in the test section was monitored, using a pressurized air-over-water manometer which was modified so that it could be inclined from a vertical position for improved accuracy during low pressure drop measurements.

The flow patterns in the tube bundles were identified by means of visual observations and photographic techniques using a 35 mm still camera and an 8 mm video-camera. The visualization of flow revealed that the air-water flow was well-mixed before it reached the first row of tubes.

The experiments were conducted at near-atmospheric pressure (101-140 kPa) under the following conditions:

 $u_{sG} = 0.047$ to 9.3 m/s, $u_{sL} = 0.0011$ to 0.65 m/s, $g_T = 1.6$ to 650 kg/(m² s), x = 0.0001 to 0.89

and

 $\epsilon = 0.11$ to 0.99;

where g_T is the total mixture mass velocity. The temperature of the mixture was about 20-30°C for all tests. The estimated uncertainty in the flow rate measurements is 2-5%.

Bypassing of the tube bundle inside the shell is usually eliminated by attaching half-tubes to the shell. Using a suggestion presented by HEDH (1983), a test section without half-tubes exposed to the flow was built. The very good agreement between the measured values of the frictional pressure drop for one-phase flow with those calculated by the new methods of Fuji & Shinzato (1980) and Gaddis & Gnielinski (1985), showed that bypassing did not occur.

PATTERN DEFINITION

Considerable differences exist between the definitions of gas-liquid two-phase flow patterns in various studies. In many instances, the definitions are not clear and comprehensive. Such a situation also prevails in the relatively well-tested case of flow in a pipe. The extent of this problem is mentioned in the works by Nguyen & Spedding (1977) and Spedding & Nguyen (1980), who point out the inconsistent nomenclature which very often makes comparison of the results of various authors difficult. They also explain the existing state caused by the occurrence of intermittent flow patterns.

The fact that flow on the shell side of a cross tube bundle has some common features with the other better-tested cases of two-phase flow is a reason for making use of those experiments. When working out the definitions of flow patterns, the following cases (according to their priority) were considered:

- (a) Studies of flow patterns in tube bundle cross-flow which have been carried out to date and, in particular, the works by Grant & Murray (1972), Kondo & Nakajima (1980) and Pettigrew *et al.* (1988).
- (b) Comparative works for flow in a pipe (Nguyen & Spedding 1977; Speeding & Nguyen 1980; Troniewski & Ulbrich 1984b).
- (c) Flow along a tube bundle (Venkateswararao et al. 1982).
- (d) Flow in rectangular channels and singularities of the flow with regard to flow in a pipe (Mishima *et al.* 1991; Troniewski & Ulbrich 1984a).
- (e) Two-phase flow in a large volume which is the shell of the apparatus. This occurs in the zone before and behind tube bundle (Sollychin *et al.* 1991).

Because flow pattern identification by means of visual observation is subjective it is essential that flow patterns should be defined in detail.

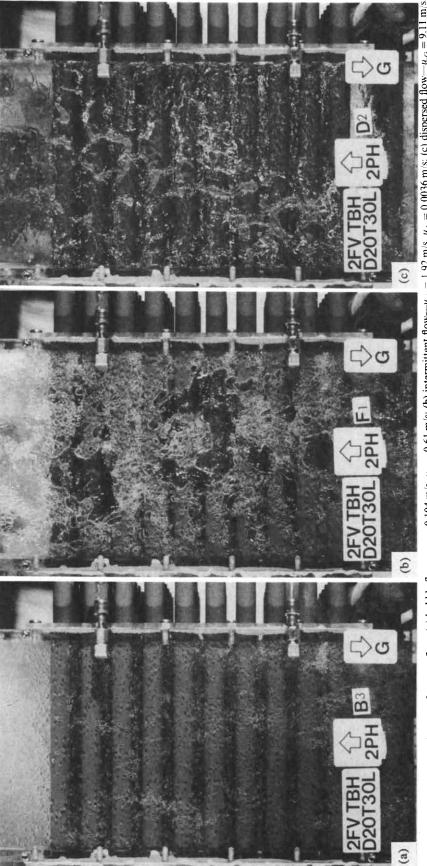


Figure 7. Flow pattern development for cross-flow: (a) bubble flow— $u_{sG} = 0.104 \text{ m/s}$, $u_{sL} = 0.61 \text{ m/s}$; (b) intermittent flow— $u_{sG} = 1.92 \text{ m/s}$, $u_{sL} = 0.0036 \text{ m/s}$; (c) dispersed flow— $u_{sG} = 9.11 \text{ m/s}$.

After studying the results cited above and after carrying out visual obserations, using photographic techniques and a video-camera, for a wide range of gas and liquid volume fluxes with the setup in figure 5 (examples of the photographs are shown in figure 7), the following classification of flow patterns was proposed:

Bubble (B)—Dispersed gas distributed as discrete small bubbles in the continuous liquid phase, whose diameters are less than the characteristic spacing between the tubes and generally uniform in size. The shape of the bubble is near eliptical. With an increase in the gas and liquid velocities, the number of bubbles grows to fill the entire channel cross-section. Eliptical bubbles are found to change as if they were pressed against the wall, as the bubble size is larger than the tube clearance. It is very important that liquid flows as the continuous phase without any local oscillation. Intermittent (I)—This flow is characterized by an irregular alternating motion of the liquid and gas. The direction of the liquid flow changes in an erratic and irregular manner from upflow to downflow and vice versa. Liquid flows downward not only as a film but also as units of liquid which occupy much of the cross-sectional area. Gas flows not only as spherically or elliptically capped bubbles but also as large flattened and irregular bubbles, whose height is several times greater than the tube diameter and their width is equivalent to the tube clearance.

Dispersed (D)—This flow is characterized by regular dispersed droplets which are carried out the gas, initially above the tube bundle, and then also between the tubes. Part of the liquid flows as irregular moving units and this flow pattern is called **intermittent-dispersed (ID)**, or liquid flow as a thin film, with surface waves occupying the tube wall or the shell wall [**annular-dispersed flow (AD)**]. The case where the entire flux of liquid flows as droplets is possible but is difficult to accomplish.

Figure 8 presents the proposed classification of flow patterns.

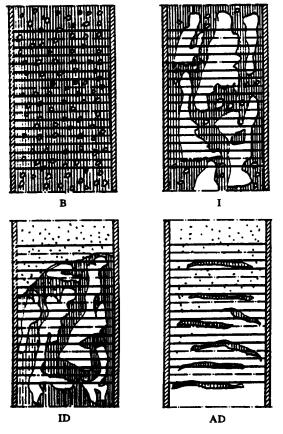


Figure 8. Flow patterns in vertical upward flow across a tube bundle.

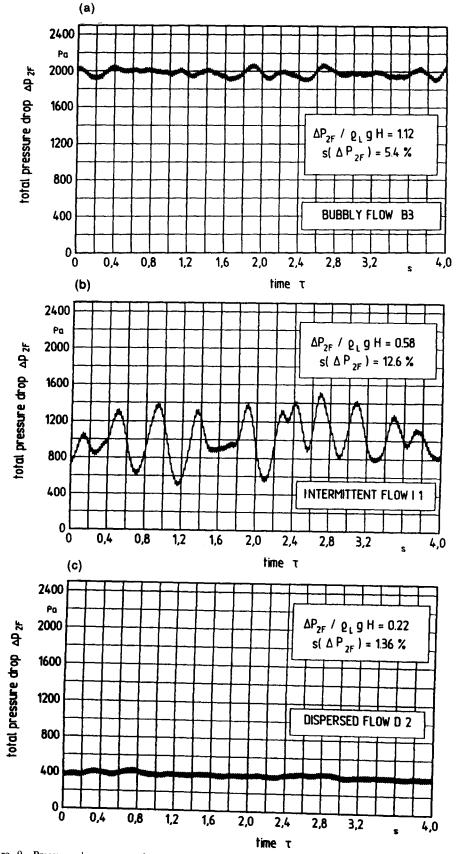


Figure 9. Pressure drop traces for a particular flow pattern: (a) bubbly flow— $u_{sG} = 0.104 \text{ m/s}$. $u_{sL} = 0.61 \text{ m/s}$; (b) intermittent flow— $u_{sG} = 1.92 \text{ m/s}$, $u_{sL} = 0.0036 \text{ m/s}$; (c) dispersed flow— $u_{sG} = 9.11 \text{ m/s}$. $u_{sL} = 0.0016 \text{ m/s}$.

The problem of the subjectivity of visual evaluation, discussed by many authors (Drahos & Cermak 1989; Ulbrich 1989) is often solved by analysis of the changes in the gas volume fraction over time or indirectly by investigations of the changes over time of parameters such as the pressure on the wall, the pressure drop, the temperature etc. (Hubbard & Dukler 1966; Drahos & Cermak 1989; Ulbrich 1989; Heck & Buchholz 1988; Hofmann & Gasche 1988).

Researchers in this area seem to have ignored the paradox that in of all the works proposing a method of objective evaluation of flow patterns, the methods were verified by comparison with the results of visual observations! It seems that such a procedure, visual observation supplemented by objective evaluation of a flow pattern is the most preferred when it is possible to apply it and, particularly, at the stage of laboratory tests.

For the needs of the present paper, the measurements of the two-phase mixture pressure drop were used as a parameter characterizing the fluctuations of the phase volume fraction. Figure 9 presents the fluctuations of the pressure drop over time for three hydrodynamic conditions, analogous to the flow patterns presented in figure 7. The nature of the variations of the pressure drop over time for an intermittent flow pattern differs distinctly from those for other patterns.

Comparing the mean value of the pressure drop $\Delta \overline{P}_{2F}$ and the standard deviation of the pressure drop $s(\Delta P_{2F})$ with the hydrostatic pressure drop for an apparatus filled only with liquid, $\rho_L gH$, where H is the height between the pressure taps, it is possible to distinguish clearly bubble and dispersive flow patterns. This way of estimating a flow pattern, when the differential pressure is normalized by the differential pressure of a static fluid column, was applied by Matsui (1984) and Tutu (1982) for two-phase flow in a vertical pipe.

RESULTS

On the basis of the classification of flow patterns presented in figure 8, a two-phase flow pattern in tube bundle cross-flow was visually estimated. The visual observation was supplemented by fixation of images by means of photographic techniques (figure 7) and a video-camera. In the case of filming it was possible to re-evaluate a flow pattern, which was very important for cases when the local velocities exceeded 1 m/s and visual estimation was very difficult. By evaluating the flow patterns, variations of the pressure drop over time were registered. As has already been mentioned, the nature of the variations of the pressure (pressure drop) over time is characteristic of a particular flow pattern (figure 9). By visual estimation of the variations of the pressure drop over time, exposed on a computer screen, and by defining basic stochastic parameters (the mean value and the standard deviation normalized by the differential pressure of the static column, the skewness and the flatness coefficient), they were finally compared with the results from visual observation of a two-phase flow pattern.

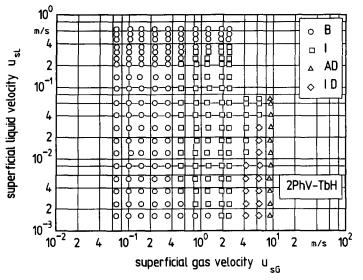


Figure 10. Observed flow patterns in the tested flow rate range.

By grading the superficial velocities of the gas and liquid in geometrical progression, equal distances between particular points were obtained on the flow pattern map (figure 10) whose coordinate axes are logarithmic.

The gas and liquid velocities are calculated in the minimum shell side cross-section (i.e. in the gap between the tubes) and although the velocities are maximal, because the phase volume fluxes concern the overall cross-section in which the mixture flows, they are still called superficial velocities.

The superficial velocities of air and water were changed in the range $u_{sG} = 0.047$ to 9.3 m/s and $u_{sL} = 0.0011$ to 0.65 m/s, respectively, to give a large area on the map for which the maximum to minimum velocity ratio is 125 for air and 400 for water.

DISCUSSION

Figure 11 compares the results of the present authors with the flow pattern map proposed by Grant & Murray (1972). All points identified as an intermittent flow pattern show agreement with the flow pattern map [table 3(a)], whereas practically all points identified as a bubbly flow pattern are situated on the flow pattern map in the region of an intermittent flow pattern. Also, a large part of the points identified as a dispersed flow pattern lies in the area of an intermittent flow pattern (more precisely, identified since visual observation was supplemented by an analysis of the fluctuation of the pressure drop) and that plotted according to the flow pattern map proposed by Grant & Murray (1972), which is an unsatisfactory result. One of the reasons for such relatively low agreement

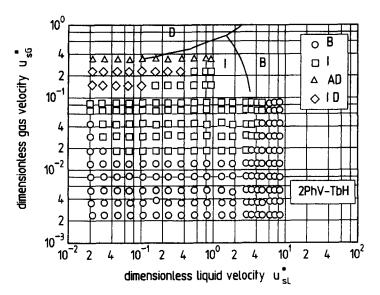


Figure 11. Comparison of the data presented in table 1 with the flow pattern map of Grant & Murray (1972).

Table 3. Statistical analysis of flow	pattern predictions for our data:	: (a) Grant & Murray	(1972); (b) Taitel et al. (1980)

(a) Data: own, TB Flow reg. r						(b) Data: own, TB Flow rep					
		E:	xp. flov	v patter	:n			E	xp. flo	w patte	rn
No. of data points		В	I	D	Sum	No. of data points		В	I	D	Sum
Pred. flow pattern	В	9	0	0	9	Pred. flow pattern	В	35	15	18	68
	I	110	80	17	207		I	84	65	6	155
	D	0	0	7	7		D	0	-0	0	0
	UN	0	0	0	0		UN	0	0	0	0
	Sum	119	80	24	223		Sum	119	80	24	223
43.0% of dat	a points	are pred	licted c	orrectly	/	44.8% of dat	a points	are pre	dicted c	orrectl	v

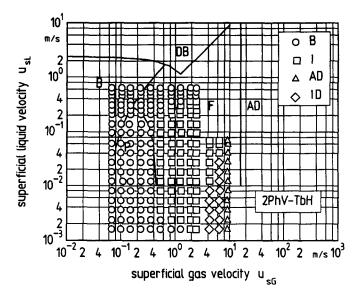


Figure 12. Comparison of the experimental data with the flow pattern map of Taitel et al. (1980).

may be the problem connected with the extrapolation of the transition lines on the flow pattern map proposed by Grant & Murray (1972), which was discussed when the flow pattern map was compared with the literature data (figure 3) and which is a serious drawback of this map (narrow range of changes of the volume fluxes of both phases).

In figure 12 the results of the present authors are compared with the map of Taitel *et al.* (1980), this was made possible by assuming that two-phase flow occurs in a rectangular channel with a hydraulic diameter equal to a double gap between the tubes and that flow patterns B and DB are for flow on the bubble pattern shell side, P and F are for intermittent flow and AD is for dispersive flow. As before, <50% of the points show agreement between the observed flow pattern and that plotted on the basis of the flow pattern map.

Since for both flow pattern maps the comparison with our results (figures 11 and 12, table 3) and with the literature data (figures 3 and 4, table 2) is not satisfactory, we decided to work out a new flow pattern map. Because of the absence of results for mixtures other than air-water, we decided to construct flow pattern maps in the most natural coordinate system whose parameters are

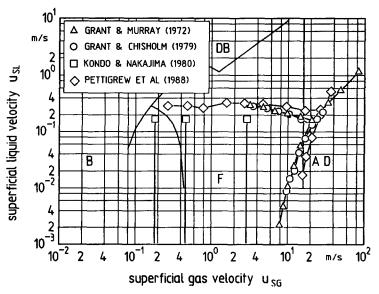


Figure 13. Comparison of the flow pattern maps for vertical upward gas-liquid two-phase flow across a tube bundle.

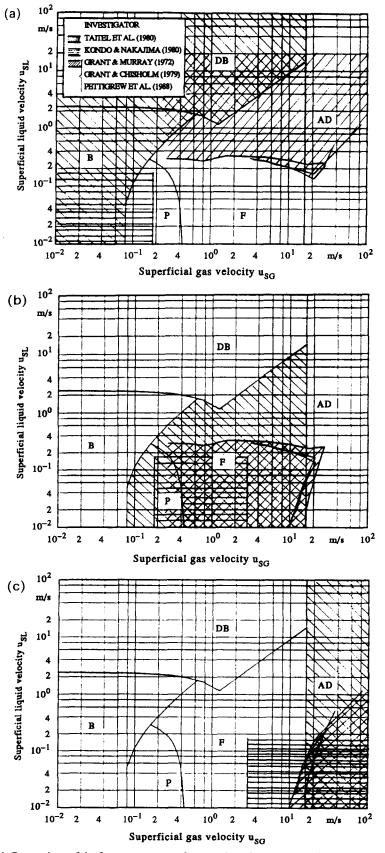


Figure 14. Comparison of the flow pattern maps for two-phase flow across a tube bundle: (a) bubble flow; (b) intermittent flow; (c) dispersed flow.

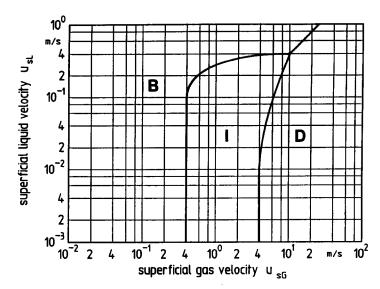


Figure 15. Generalized flow pattern map (B-bubble, I-intermittent, D-dispersed flows).

the superficial velocities of both phases. Attempts to include the influence of physical parameters, such as the gas density ρ_G , the liquid density ρ_L , the gas viscosity η_G , the liquid viscosity η_L or the surface tension σ , upon the position of the transition lines of the flow pattern map for tube bundle cross-flow were restricted to the corrections determined by Baker (1954), despite the fact that his work concerned gas-liquid two-phase flow only in a horizontal pipe.

Figure 13 compares, after transformation on the flow pattern map proposed by Taitel *et al.* (1980) (since the map was worked out on the basis of theoretical considerations it is regarded as a standard and is practically a reference point for all the works published after 1980) the flow pattern maps, quoted in the literature, for vertical tube bundle cross-flow.

The three flow pattern maps proposed by Grant & Murray (1972), Grant & Chisholm (1979) and Pettigrew *et al.* (1988) are practically identical and they differ only in the coordinate system used. The coordinate system used in the work by Grant & Chisholm (1979) is commonly applied, including the case of horizontal flow (Grant & Murray 1974), whereas Pettigrew *et al.* (1988) transposed the map proposed by Grant & Murray (1972) into another system and, in addition, verified the position of the boundary between bubble and intermittent flow.

For a quite different range of gas and liquid superficial velocities, for which Grant & Murray found an unstable regime in the laboratory exchanger tested by them, Kondo & Nakajima (1980)

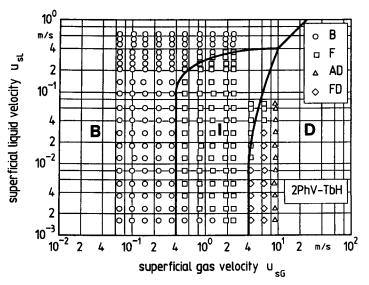


Figure 16. Comparison of the data presented in table 1 with the proposed flow pattern map.

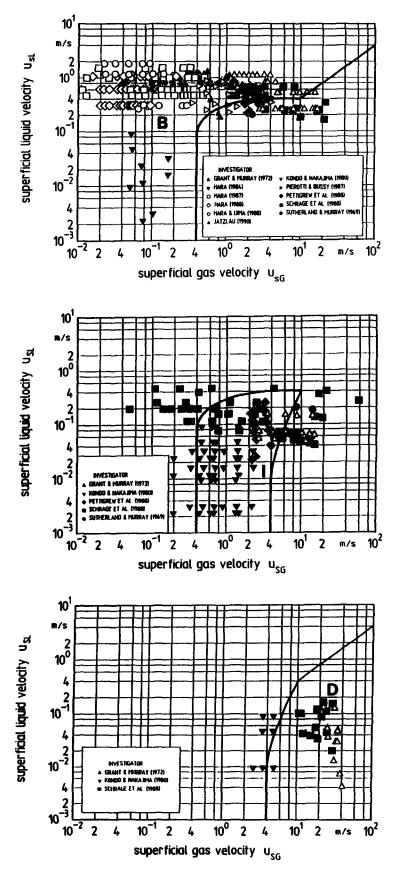


Figure 17. Comparison of the experimental data with the proposed flow pattern map.

					(b) Data: total Flow	v reg. m	ap: this	paper		
	Ex	p. flow	v patter	m			E	xp. flov	v patte	rn
	В	I	D	Sum	No. of data points		В	I	D	Sum
B	115	1	0	116	Pred. flow pattern	В	467	22	0	489
I D	4	2	0	81 26		I D	28 14	21	6 25	118 60
UN	0	0	0	0		UN	0	0	0	0 667
-	eg. ma B I D UN	eg. map: this pa Ex B 115 I 4 D 0 UN 0	eg. map: this paper Exp. flow B I B 115 1 I 4 77 D 0 2 UN 0 0	Exp. flow patter B I D B 115 1 0 I 4 77 0 D 0 2 24 UN 0 0 0	eg. map: this paper Exp. flow pattern B I D Sum B 115 1 0 116 I 4 77 0 81 D 0 2 24 26 UN 0 0 0 0	eg. map: this paperFlowExp. flow patternBIDSumNo. of data pointsB11510116Pred. flow patternI477081Pred. flow patternD022426UNUN00000	eg. map: this paperFlow reg. mExp. flow patternBIDSumNo. of data pointsB11510116Pred. flow patternBI477081ID022426DUN0000UN	eg. map: this paperFlow reg. map: thisExp. flow patternEBIDSumNo. of data pointsBB11510116Pred. flow patternB 467 I477081I28D022426D14UN0000UN0	Flow reg. map: this paper Flow reg. map: this paper Exp. flow pattern Exp. flow B I D Sum B 115 1 0 116 I 4 77 0 81 D 0 2 24 26 UN 0 0 0 0	Flow reg. map: this paperExp. flow patternExp. flow patternBIDSumB11510I4770D02242426DI142125

Table 4. Statistical analysis of flow pattern predictions with the proposed flow pattern map: (a) our data, (b) data listed in table 1

conducted experiments and obtained results different from those presented by others. It is interesting, however, that the flow pattern map proposed by Kondo & Nakajima (1980) is similar to a certain extent to that presented by Taitel *et al.* (1980), and the ranges of occurrence of plug and froth flow patterns are covered quite well. Still further information is supplied by comparison of the ranges of occurrence of particular flow patterns, which are presented in figure 14. Although it is impossible to determine their boundaries exactly, it is astonishing to notice the general tendency that the supplementary data, given by Grant & Murray (1972) and Kondo & Nakajima (1980), in the sense of the regions of occurrence of particular flow patterns, are of similar shape to on the map presented by Taitel *et al.* (1980).

After analysing the results of our research (figure 10) and including the results of the comparison of flow pattern maps (figures 13 and 14), we propose the flow pattern map presented in figure 15.

In figures 16 and 17 the results of our experiments are compared with data from the literature using a general flow regime map. The very good agreement of our data is evident [table 4(a)], also high agreement with the literature data listed in table 1, estimated as >85% [table 4(b)], is an objective evaluation of the accuracy of the proposed flow pattern map. It is only proper to point out that agreement in the range of 85% is very seldom achieved for two-phase flow in a pipe and a similar result was accepted as satisfactory by McQuillan & Whalley (1983).

In order to propose a general flow regime map, in figure 18(a) the transition lines for bubble flow are compared with various flow pattern maps presented by:

- Grant & Murray (1972), for tube bundle cross-flow.
- Pettigrew *et al.* (1988), for tube bundle cross-flow, assuming that the boundary between bubble and intermittent flow appears for a homogeneous gas void fraction of $\epsilon = 0.9$.
- Taitel *et al.* (1980), Mishima & Ishii (1984) and Barnea (1986), for chosen theoretical analyses for two-phase flow in a pipe, after assuming that tube bundle cross-flow occurs in a rectangular channel with a hydraulic diameter equal to double the gap between the tubes.
- Spedding & Nguyen (1980), for flow in a vertical pipe. These authors treated the problem of intermittent flow very scrupulously.

The transition lines for dispersive flow are compared in figure 18(b) with various flow pattern maps:

- Hofman & Gasche (1988), for bubbly column flow, where the boundary is not sharp but is presented as a zone.
- Mishima et al. (1991), for vertical flow in a narrow rectangular duct.
- Grant & Murray (1972), for tube bundle cross-flow.
- Taitel et al. (1980), Mishima & Ishii (1984) and Barnea (1986) [as in figure 18(a)].

Although complete agreement is not achieved, and it would be difficult to evaluate this comparison with respect to quantity, the general course of the boundary lines is maintained.

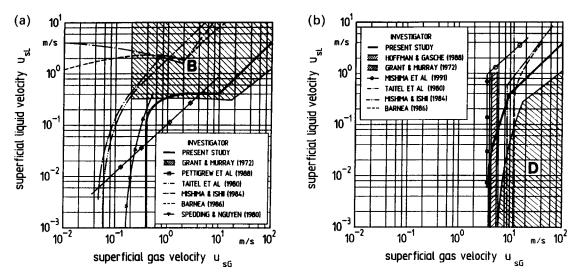


Figure 18. Comparison of the predictions of the present work with other flow pattern maps: (a) the transition to bubble flow; (b) the transition to dispersed flow.

CONCLUSIONS

The flow pattern map, proposed on the basis of our results and the analysis of flow pattern maps cited in the literature, correlates well with the literature data presented in table 1.

One should remember that a tube bundle is usually placed inside a cylindrical shell, whose length is often close to the dimensions of a channel cross-section with regard to the flow. Thus, these conditions stray considerably from the situation which takes place in two-phase flow in a pipe. Frequently, the initial conditions of the flow, before and behind the tube bundles, can exert an essential influence on the formation of the flow patterns. These factors and the multi-parameter geometry of tube bundles (pipe diameter, pitch, number of rows, tube bundle arrangement, tube bundle orientation) still require much further study.

Acknowledgements—This work was performed by R. Ulbrich at the Institute of Process Engineering of the University of Hannover under a research grant awarded by the Alexander von Humboldt Foundation. R. Ulbrich wishes to express his appreciation for the invitation to spend his scholarship in Germany.

REFERENCES

- BAKER, O. 1954 Simultaneous flow of oil and gas. Oil Gas J. 53, 185-195.
- BARNEA, D. 1986 A unified model for predicting flow pattern transitions for the whole range of pipe inclinations. Int. J. Multiphase Flow 13, 1-12.
- BELL, K. J., TABOREK, J. & FENOGLIO, F. 1970 Interpretation of horizontal intube condensation heat transfer correlations with a two-phase flow regime map. *Chem. Engng Progr. Symp. Ser.* **66**(102), 150-165.
- BERGLES, A. E., ROOS, J. P. & BOURNE, J. G. 1968 Investigation of boiling flow regimes and critical heat flux. Report NYO-3304-13, Dynatech Corp., Cambridge, MA.

CHISHOLM, D. 1983 Two-phase Flow in Pipelines and Heat Exchangers. Georg Godwin, London.

- DOWLATI, R., KAWAJI, M. & CHAN, A. M. C. 1990a Pitch-to-diameter effect on two-phase flow across an in-line tube bundle. AIChE JI 36, 765-772.
- DOWLATI, R., CHAN, A. M. C. & KAWAJI, M. 1990b Comparison of void fraction and friction pressure drop in adiabatic two-phase flow between a horizontal in-line and staggered tube bundle. In *Proc. 25th Mtg of the Eur. Two-phase Flow Group*, Varese, pp. 305-311.
- DRAHOS, J. & CERMAK, J. 1989 Diagnostics of gas-liquid flow patterns in chemical engineering systems. Chem. Engng Process. 26, 147-164.
- FUJI, T. & SHINZATO, K. 1980 On pressure coefficients of tube banks in crossflow. Private communication.

- GADDIS, E. S. & CNIELINSKI, V. 1985 Pressure drop in cross flow across tube bundles. Int. Chem. Engng 25, 1–15.
- GRANT, I. D. R. 1973 Flow and pressure drop with single-phase and two-phase flow on the shell-side of segmentally balfled shell-and-tube heat exchangers. In *Advances in Thermal and Mechanical Design of Shell-and-tube Heat Exchangers*. Report NEI-590, National Engineering Lab.
- GRANT, I. D. R. & CHISHOLM, D. 1979 Two-phase flow on the shell side of a segmentally baffled shell-and-tube heat exchanger. *Trans. ASME Jl Heat Transfer* 101, 38–42.
- GRANT, I. D. R. & MURRAY, I. 1972 Pressure drop on the shell-side of a segmentally baffled shell-and-tube heat exchanger with vertical two-phase flow. Report NEL-500, National Engineering Lab.
- GRANT, I. D. R. & MURRAY, I. 1974 Pressure drop on the shell-side of a segmentally baffled shell-and-tube heat exchanger with horizontal two-phase flow. Report NEL-560, National Engineering Lab.
- HAQUET, J. F., GOUIRAND, J. M., MARET, P. & IVARS, J. F. 1991 An application of advanced optical probe instrumentation in the local tube bundle subchannel anlaysis. Presented at the *Eur. Two-phase Group Mtg*, Rome, Paper L3.
- HARA, F. 1984 Air-bubble effects on vortex-induced vibrations of a circular cylinder. In *Proc. Symp. on Flow Induced Vibrations, ASME*, New Orleans, pp. 103-113.
- HARA, F. 1987 Vibration of a single row of circular cylinders subjected to two-phase bubble cross-flow. In *Proc. Int. Conf. on Flow Induced Vibrations*, Bowness-on-Windermere, U.K., pp. 203-208.
- HARA, F. 1988 A flow visualisation study of a single row of circular cylinders vibrating in water and two-phase crossflows. In Proc. Int. Symp. on Flow Induced Vibrations and Noise, Chicago, IL, pp. 77-89.
- HARA, F. & IJIMA, T. 1988 Vibration of two circular cylinders in tandem subjected to two-phase bubble cross flows. In Proc. Int. Symp. on Flow Induced Vibrations and Noise, Chicago, IL, pp. 63–78.
- HECK, J. & BUCHHOLZ, R. 1988 Measuring two-phase flow in bubble columns. Paper presented at the German/Japanese Symp. on Bubble Columns, Schwerte.
- HEDH 1983 Heat Exchanger Design Handbook. Hemisphere, New York.
- HOFMANN, H. & GASCHE, H. E. 1988 Investigations of the fluid-dynamics of bubbly columns. Paper presented at the German/Japanese Symp. on Bubble Columns, Schwerte.
- HUBBARD, N. G. & DUKLER, A. E. 1966 The characterization of flow regimes for horizontal two-phase flow. In *Proc. 1966 Heat Transfer and Fluid Mechanics Institute*, pp. 100-121. Stanford Univ. Press, CA.
- ISHIHARA, K., PALEN, J. W. & TABOREK, J. 1980 Critical review of correlations for predicting two-phase flow pressure drop across tube banks. *Heat Transfer Engng* 1, 23-32.
- JATZLAU, B. 1990 Schwingungsanregungen in zweiphasig durchstroemten, unbeheizten und beheizten Rohrbuendeln. Dissertation, Tech. Univ. München.
- KONDO, M. & NAKAJIMA, K. I. 1980 Experimental investigation of air-water two-phase upflow across horizontal tube bundles. Part I—Flow pattern and void fraction. *Bull. JSME* 23, 385-393.
- KORTE, H. J., STEIFF, A. & WEINSPACH, P. M. 1988 Heat transfer in bubble columns. Paper presented at the German/Japanese Symp. on Bubble Columns, Schwerte.
- LOCKHART, R. W. & MARTINELLI, R. C. 1949 Proposed correlation of data for isothermal two-phase two-component flow in pipes. *Chem. Engng Progr.* 45, 39-48.
- MATSUI, G. 1984 Identification of flow regimes in vertical gas-liquid two-phase flow using differential pressure fluctuations. Int. J. Multiphase Flow 10, 711-720.
- McQuillan, K. W. & Whalley, P. B. 1983 Flow patterns in vertical two-phase flow. Report AERE-R 11032, AERE Harwell, U.K.
- MISHIMA, K. & ISHII, M. 1984 Flow regime transition criteria for upward two-phase flow in vertical tubes. Int. J. Heat Mass Transfer 27, 723-737.
- MISHIMA, K., HIBIKI, T. & NISHIHARA, H. 1991 Some characteristics of gas-liquid flow in narrow rectangular ducts. In *Proc. Int. Conf. on Multiphase Flows*, Tsukuba, pp. 485-488.

- MULLER-STEINHAGEN, H. & HECK, K. 1986 A simple friction pressure drop correlation for two-phase flow in pipes. Chem. Engng Process. 20, 297-308.
- NGUYEN, V. T. & SPEEDING, P. L. 1977 Holdup in two-phase gas-liquid flow. Chem. Engng Sci. 32, 1003-1014.
- PETTIGREW, M. J., TAYLOR, C. E. & KIM, B. S. 1988 Vibrations of tube bundles in two-phase cross flow: part 1—hydrodynamic mass and damping. In *Proc. Int. Symp. on Flow Induced Vibrations and Noise*, Chicago, IL, pp. 79–103.
- PIEROTTI, G. & BUSSY, B. 1987 Experimental studies of two-phase flow across tube banks. Presented at the Eur. Two-phase Flow Group Mtg, Trondheim, Paper H2.
- SCHRAGE, D. S., HSU, J. T. & JENSEN, M. K. 1988 Two-phase pressure drop in vertical crossflow across a horizontal tube bundle. AIChE Jl 34, 107-115.
- SOLLYCHIN, R., GARLAND, W. J. & CHANG, J. S. 1991 Analysis of flow regimes in a vertical two-phase column and modelling of their transitions. In *Proc. Int. Conf. on Multiphase Flows*, Tsukuba, pp. 21–24.
- SPEDDING, P. L. & NGUYEN, V. T. 1980 Regime maps for air water two phase flow. Chem. Engng Sci. 35, 779-793.
- STEIFF, A., SCHLUTER, S. & WEINSPACH, P. M. 1991 Heat transfer in two and three-phase bubble columns with and without installations. In *Proc. 2nd Japanese/German Symp. on Bubble Columns*, Kyoto, pp. 31-36.
- STOMMA, Z. 1979 Two-phase flows- void fraction values determination. Report INR/1818/IX/R/A, Inst. of Nuclear Reserach, Swierk/Warszawa.
- SUTHERLAND, L. A. & MURRAY, I. 1969 Pressure drop and heat transfer on the shell-side of a model heat exchanger with two-phase flow. Report NEL-395, National Engineering Lab.
- TAITEL, Y., BARNEA, D. & DUKLER, A. E. 1980 Modelling flow pattern transitions for steady upward gas-liquid flow in vertical tubes. AIChE Jl 26, 345-354.
- TRONIEWSKI, L. & ULBRICH, R. 1984 Two-phase gas-liquid flow in rectangular channels. *Chem. Engng Sci.* 39, 751-765.
- TRONIEWSKI, L. & ULBRICH, R. 1984 The analysis of flow regime maps of two-phase gas-liquid flow in pipes. Chem. Engng Sci. 39, 1213-1227.
- TUTU, N. K. 1982 Pressure fluctuations and flow pattern recognition in vertical two phase gas-liquid flows. Int. J. Multiphase Flow 8, 443-447.
- ULBRICH, R. 1989 Identification of gas-liquid flow in a channel. Papers of Opole Technical University, No. 32.
- VDI 1977 VDI-W. VDI-Verlag, Dusseldorf.
- VENKATESWARARAO, P., SEMIAT, R. & DUKLER, A. E. 1982 Flow pattern transition for gas-liquid flow in a vertical rod bundle. Int. J. Multiphase Flow 8, 509-524.
- WILLIAMS, C. L. & PETERSON, A. C. JR 1978 Two-phase flow patterns with high-pressure water in a heated four-rod bundle. *Nucl. Sci. Engng* 68, 155-169.